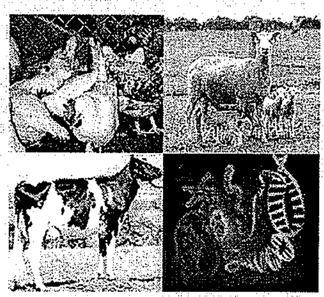
BLUP animal model



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UNE

IAEA, Korea, April, 2006

Why BLUP?

BLUP predicts breeding value more accurately

- BLUP corrects for environmental deviations due to fixed effects
- BLUP uses all pedigree information: accounts for selection and genetic trends etc.

BLUP EBV can be estimated for animals without phenotypes using information from their relatives

....

BLUP animal model

Mixed linear model

$$y = X\beta + Za + e$$

y - Phenotypic value $E(y) = X\beta$

β - Fixed effects

a - Random genetic effects ~ $(0, \sigma_a^2)$

e - Residuals ~ $(0, \sigma_e^2)$

X , Z - Incidence matrices

BLUP animal model

Mixed linear model

$$y = X\beta + Za + e$$

Variance covariance of *y* is,

$$V = ZAZ'\sigma_a^2 + I\sigma_e^2$$

A - numerator relationship matrix

I - identity matrix

BLUP animal model

Mixed linear model

$$y = X\beta + Za + e$$

$$\text{BLUP }(a) = \hat{a} = AZ'\sigma_a^2 V^{-1}(y - X\hat{\beta})$$

$$\text{BLUE }(\beta) = \hat{\beta} = (X'V^{-1}X)^{-1}X'V^{-1}y$$

These solutions are important

in genetic evaluation and breeding program

Mixed linear model

$$y = X\beta + Za + e$$

$$\text{BLUP }(a) = \hat{a} = AZ'\sigma_a^2 V^{-1}(y - X\hat{\beta})$$

$$\text{BLUE }(\beta) = \hat{\beta} = (X'V^{-1}X)^{-1}X'V^{-1}y$$

These solutions can be obtained by mixed model equation (MME)

$$\begin{bmatrix}\hat{\beta}\\ \hat{a}\end{bmatrix} = \begin{bmatrix}X'X & X'Z\\ Z'X & Z'Z + A^{-1}\alpha\end{bmatrix}^{-1}\begin{bmatrix}X'y\\ Z'y\end{bmatrix} \qquad (\alpha = \frac{\sigma_e^2}{\sigma_a^2} = \frac{1-h^2}{h^2})$$

BLUP using MME

No need to invert V

- Usually computationally efficient

Can obtain $\hat{a}$ and $\hat{\beta}$ simultaneously

---- *widely used for genetic evaluation,* ----

BLUP animal model

Example

Herd	Animal	Sire	Dam	Phenotype
1	1	0	0	100
2	2	0	0	130
1	3	0	0	120
2	4	1	2	110
1	5	4	3	140

We want to estimate herd effects ($\hat{\beta}$) and random genetic animal effects ($\hat{a}$)

BLUP animal model

Example

Herd	Animal	Sire	Dam	Phenotype
1	1	0	0	100
2	2	0	0	130
1	3	0	0	120
2	4	1	2	110
1	5	4	3	140

Constructing X

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

BLUP animal model

Example

Herd	Animal	Sire	Dam	Phenotype
1	1	0	0	100
2	2	0	0	130
1	3	0	0	120
2	4	1	2	110
1	5	4	3	140

Constructing Z

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

BLUP animal model

Example

Herd	Animal	Sire	Dam	Phenotype
1	1	0	0	100
2	2	0	0	130
1	3	0	0	120
2	4	1	2	110
1	5	4	3	140

Constructing A (NRM)

$$\begin{bmatrix} 1 & 0 & 0 & 0.5 & 0.25 \\ 0 & 1 & 0 & 0.5 & 0.25 \\ 0 & 0 & 1 & 0 & 0.5 \\ 0.5 & 0.5 & 0 & 1 & 0.5 \\ 0.25 & 0.25 & 0.5 & 0.5 & 1 \end{bmatrix}$$

NRM

If both parents (*s* and *d*) of animal *i* are known

$$r_{ij} = r_{ji} = 0.5(r_{js} + r_{jd}) \qquad j = 1 \sim (i-1)$$
$$r_{ii} = 1 + 0.5(r_{sd})$$

If only one parent (*s*) of animal *i* is known

$$r_{ij} = r_{ji} = 0.5(r_{js}) \qquad j = 1 \sim (i-1)$$
$$r_{ii} = 1$$

If both parents (*s* and *d*) of animal *i* are unknown

$$r_{ij} = r_{ji} = 0 \qquad j = 1 \sim (i-1)$$
$$r_{ii} = 1$$

Example

Herd	Animal	Sire	Dam	Phenotype
1	1	0	0	100
2	2	0	0	130
1	3	0	0	120
2	4	1	2	110
1	5	4	3	140

Constructing V

$$\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0.5&0.25\\0&1&0&0.5&0.25\\0&0&1&0&0.5\\0.5&0.5&0&1&0.5\\0.25&0.25&0.5&0.5&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}\times\sigma_a^2+\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}\times\sigma_e^2$$

Example

Herd	Animal	Sire	Dam	Phenotype
1	1	0	0	100
2	2	0	0	130
1	3	0	0	120
2	4	1	2	110
1	5	4	3	140

After constructing the matrices, it is straightforward to obtain BLUE and BLUP

$$\hat{\beta} = (X'V^{-1}X)^{-1}X'V^{-1}y$$

$$\hat{a} = AZ'\sigma_a^2 V^{-1}(y - X\hat{\beta})$$

Example (using MME)

Herd	Animal	Sire	Dam	Phenotype
1	1	0	0	100
2	2	0	0	130
1	3	0	0	120
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1	5	4	3	140

Constructing X'X

$$\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

Example (using MME)

Herd	Animal	Sire	Dam	Phenotype
1	1	0	0	100
2	2	0	0	130
1	3	0	0	120
2	4	1	2	110
1	5	4	3	140

Constructing X'Z

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Example (using MME)

Herd	Animal	Sire	Dam	Phenotype
1	1	0	0	100
2	2	0	0	130
1	3	0	0	120
2	4	1	2	110
1	5	4	3	140

Constructing Z'X

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Example (using MME)

Herd	Animal	Sire	Dam	Phenotype
1	1	0	0	100
2	2	0	0	130
1	3	0	0	120
2	4	1	2	110
1	5	4	3	140

Constructing Z'Z

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Example (using MME)

Herd	Animal	Sire	Dam	Phenotype
1	1	0	0	100
2	2	0	0	130
1	3	0	0	120
2	4	1	2	110
1	5	4	3	140

Constructing MME

$$\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} + A^{-1}\alpha$$

BLUP animal model

Example

Herd	Animal	Sire	Dam	Phenotype
1	1	0	0	100
2	2	0	0	130
1	3	0	0	120
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After constructing the MME, it is straightforward to obtain BLUE and BLUP

$$\begin{bmatrix} \hat{\beta} \\ \hat{a} \end{bmatrix} = \begin{bmatrix} X'X & X'Z \\ Z'X & Z'Z + A^{-1}\alpha \end{bmatrix}^{-1} \begin{bmatrix} X'y \\ Z'y \end{bmatrix} \qquad \left(\alpha = \frac{\sigma_e^2}{\sigma_a^2} = \frac{1-h^2}{h^2}\right)$$

BLUP accuracy and response

$$C=\begin{bmatrix} X'X & X'Z \\ Z'X & Z'Z+A^{-1}\alpha \end{bmatrix}^{-1} \qquad (\alpha=\frac{\sigma_e^2}{\sigma_a^2}=\frac{1-h^2}{h^2})$$

Let the diagonal for animal *i* be C^{ii}

The accuracy of the EBV= $\sqrt{(1-C^{ii}\alpha)}$

The selection response $R=i\sigma_{EBV}$

Practical session

We will try

1. Simulating pedigree and phenotypic data
2. Estimating GLS and BLUP solutions with true h2

Compare TBV and EBV

- with varying herd effects
- without pedigree
- without herd record
- with wrong pedigree

Compare MME method and iterative BLUP

Selection response from BLUP or from phenotypes

BLUP animal model

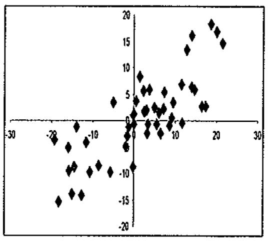
Practical session

If you are keen to learn more genetic analysis,

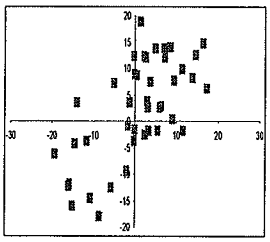
- Estimate V_a, V_e and h^2 with simulated data
  - Standard error of estimates
- Compare EBV with estimated and true h^2
- Family design for better estimating h^2
  - A few large families or many small families
- More replicates would give more reliable results

BLUP animal model

Simulation study:
Correlation between TBV and EBV (phenotypes)



BLUP solutions (r=0.83)

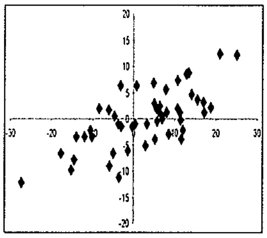


Phenotypes (r=0.81)

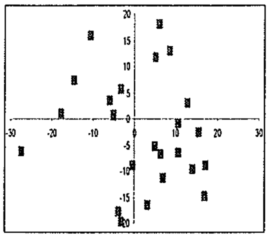
When herd effect is 0

BLUP animal model

Simulation study:
Correlation between TBV and EBV (phenotypes)



BLUP solutions (r=0.7)

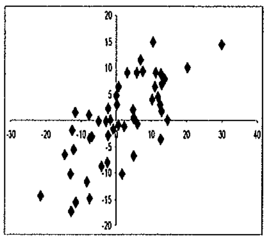


Phenotypes (r=0.2)

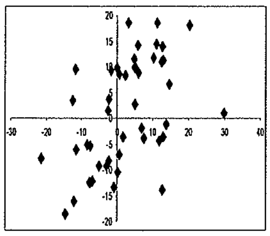
When herd effect is $\sim 2\sigma_P$

BLUP animal model

Simulation study:
Correlation between TBV and EBV



Considering herd (r=0.73)

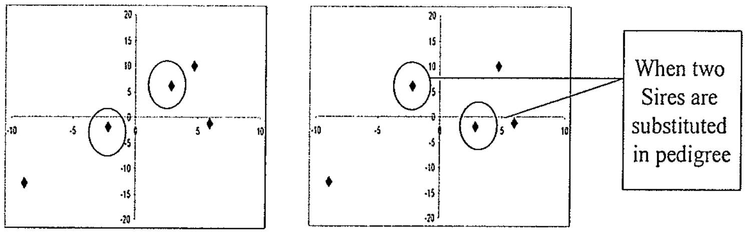


Without considering herd (r=0.56)

Herd effect should be corrected for BLUP estimation.
Otherwise, they are confounded.

BLUP animal model

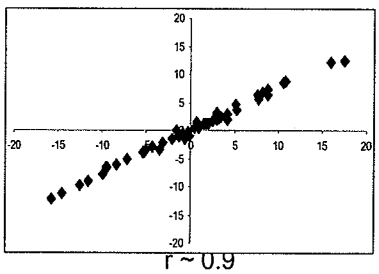
Simulation study:
Correlation between TBV and EBV



Correct pedigree (r=0.82) Wrong pedigree (r=0.63)

Pedigree record is important for BLUP estimation

Estimated h^2 and true h^2 for BLUP solution



r ~ 0.9

Correlation of BLUP solutions with estimated h^2 and true h^2 is very high